TABLE 2. POWER LAW CONSTANTS

Fluid	80°F.		95°F.	
	n	*a*	*n*	*a*
Distilled water	1.00	0.0085	1.00	0.0072
CMC domestic	0.87	0.050	0.84	0.056
CMC distilled	0.65	0.56	0.65	0.49
Nat domestic	0.89	0.040	0.91	0.030
Nat distilled	0.90	0.034	0.90	0.030
J-2P domestic	0.95	0.023	0.93	0.023
J-2P distilled	0.93	0.027	0.91	0.027

ditive were weighed to 0.1% accuracy after the moisture content of the additive had been accurately determined. The powder was dispersed in a vortex formed by agitating the water with a propeller mixer and allowed to dissolve at room temperature.

Two separate tests utilizing commercially available equipment were required to cover the shear rate range desired. The Fann Model 35 rotational viscometer was used to measure the shear stress at discrete shear rates below 3,300 sec.$^{-1}$, while the Merrill Brookfield High Shear Rate Viscometer was used in the shear stress range from 3,000 sec.$^{-1}$ to 70,000 sec.$^{-1}$.

In addition to measuring the final (after 24 hr.) viscosity, the Fann viscometer was used to determine the viscosity of each of the six solutions as a function of time.

Figure 1 demonstrates the marked dependence of the final viscosity of the CMC solution on solvent purity. Although the impurity content of the domestic water used for the tests was extremely high, there is every reason to believe that smaller amounts of impurities would still produce some deviation from the viscosity expected with the distilled water, not necessarily in proportion, however. The final viscosity of the other two additive solutions has relatively little dependence on the solvent purity, as shown in Table 2.

The effect of increased solvent purity on Natrosol was an increase in the time required for the solution viscosity to reach its steady state value. During this time the exponent *n* did not change, although the coefficient increased significantly. Fourteen minutes after mixing, the coefficient for the Natrosol distilled water solution was 88% of the final value, while the coefficient for the Natrosol domestic water solution was 98% of its final value. The steady state viscosity was reached after approximately 30 min. Neither CMC nor J-2P showed any significant difference in the time required for the viscosity to reach its final value. Of the three additives only J-2P had a viscosity independent of both time and solvent purity.

Tabulated in Table 2 are the power law constants *n* and *a* at 80° and 95°F. In every instance the effect of the additive is to alter the normal dependence of the viscosity of water on temperature.

The results of these simple experiments, shown in Figure 1 and Table 2, demonstrate that solvent purity is of considerable importance in determining the viscosity of non-Newtonian fluid solutions especially for anionic polymers. Once the rheological properties are completely defined and measured, a more realistic approach can be made to understanding the total effects of solvent purity on the rheology of an elasticoviscous fluid.

ACKNOWLEDGMENT

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LITERATURE CITED

1. Merrill, E. W., *Ind. Eng. Chem.*, **51**, 868 (1961).
2. Porter, R. S., and J. F. Johnson, *J. Appl. Phys.*, **32**, 2326 (1961).
3. Bogue, D. C., and A. B. Metzner, *Ind. Eng. Chem. Fundamentals,* **2**, 143 (1963).
4. "Handbook of Fluid Dynamics," Chapt. 7, McGraw-Hill, New York (1961).
5. Wells, C. S., *LTV Res. Center Rept. 0-71000/3R-28* (1963).
6. Shaver, R. G., and E. W. Merrill, *A.I.Ch.E. J.*, **5**, 181 (1959).
7. Metzner, A. B., and J. C. Reed, *ibid.*, **1**, 434 (1955).
8. Dodge, D. W., and A. B. Metzner, *ibid.*, **5**, 189 (1959).
9. Ripkin, J. F., and M. Pilch, *St. Anthony Falls Hydraulic Lab. Tech. Paper 42, Ser. B* (1963).
10. Oldroyd, J. B., *Proc. Roy. Soc.* (*London*), **A245**, 278 (1958).
11. Walters, K., *Quart. J. Mech. Appl. Math.*, **15**, 63 (1962).
12. Coleman, B. D., and W. Noll, *Ann. N.Y. Acad. Sci.*, **672** (1961).
13. Oldroyd, J. G., *Proc. Roy. Soc.* (*London*), **A200**, 523 (1949).

Heat Transfer Efficiency in Rough Pipes at High Prandtl Number

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Recently, Dipprey *(1)* has suggested that heat transfer efficiency, defined by heat transmission per unit pressure drop, might be higher in rough tubes than in smooth ones for fluids with a high Prandtl number. If for these fluids the increase in efficiency is sufficiently large, the use of rough pipes for transferring heat becomes economically feasible.

To date very little data have been published for heat transfer in rough pipes for fluids with Prandtl number greater than 8, although many studies have been made in the field *(1* to *3)*. Without exception previous results have shown that smooth pipes are more efficient than rough ones. Preliminary experimental results reported here, however, show a significant improvement in efficiency of heat transfer to a fluid having a Prandtl number of 349 over that obtained under similar flow conditions in a smooth tube.

EXPERIMENTAL

Three experimental test sections were used in the investigation, one being a smooth tube that was used for comparative purposes. The other two sections were roughened by internally threading two ⅜ in. I.D. brass pipes with specially made taps. The test section of the apparatus was electrically heated and the readings from the voltmeter and ammeter were used to determine the heat input. The wall temperatures were measured with four thermocouples embedded in the pipe, and the readings from them were recorded on a strip chart potentiometer. Water and Ucon, a polyalkylene glycol with a Prandtl number of 349, were used as the experimental fluids. The heat transfer coefficient and the friction factor were determined at various Reynold numbers for all three tubes from the pressure drop and temperature readings.

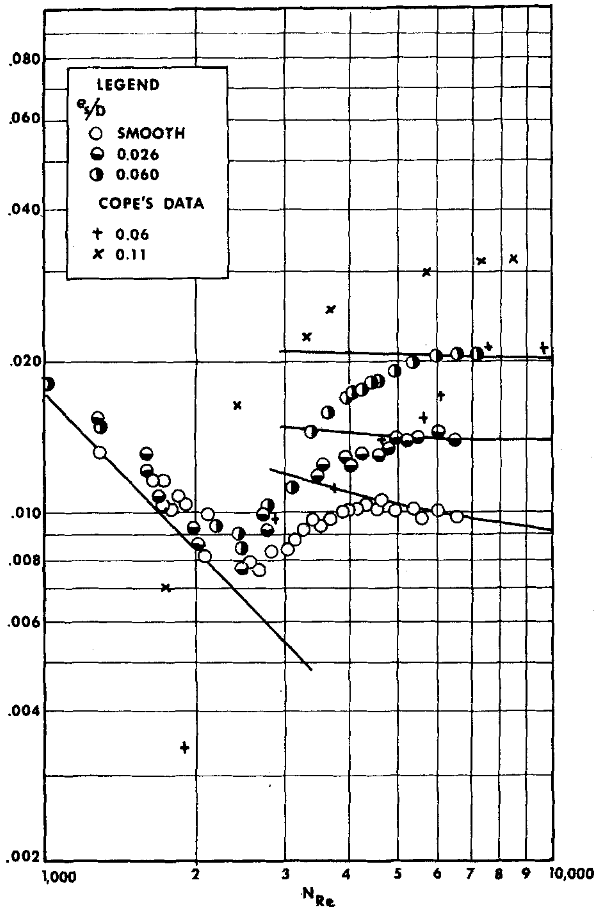

Fig. 1. Friction factor plot for smooth and rough pipes.

RESULTS

A Reynolds number range between 1,000 and 10,000 was studied; unfortunately larger values could not be obtained in the present apparatus with the highly viscous Ucon.

The friction factor f was calculated and plotted against Reynolds number as shown in Figure 1 for both the smooth and rough tubes. The experimental points for the smooth tube follow the accepted curve, and therefore support the assumption that Ucon is completely Newtonian. The equivalent sand roughness e_s/D was obtained by comparing the rough tube friction factor plots of Figure 1 with a Fanning plot (*6*). The recorded e_s/D is therefore an equivalent sand roughness and not the actually measured e/D. Initially, the smooth tube exchanger was tested by using water as the heat transfer fluid. The results agree well with the accepted data published by McAdams (*4*). Since the transfer of heat to rough pipes is affected by the Prandtl number of the fluid and the type of roughness as well as the roughness height, it is difficult to compare data with those of other workers. However, Cope (*2*) performed similar heat transfer experiments in 1941 with pipes that were roughened by knurling. Some of his data are plotted in Figures 1 and 2, in order to compare the friction factors and Nusselt numbers with the author's results. In the present investigation the bulk Prandtl number was kept constant by varying the heat flux, but in Cope's work, heat flux, wall temperatures, and the Prandtl number were varied. This accounts for the slight difference in slope between the two sets of data in Figure 2. The difference in magnitude of the Nusselt number can be attributed to the different roughness pattern and the smaller Prandtl number (approximately 7 to the authors' 8). The rough-pipe heat transfer experiments of Dipprey (*1*) and Hastrup (*3*) are also of interest but their results were obtained only with water in a different Reynolds number range (10,000 to 1,000,000).

Figure 2 clearly indicates that the increase in heat transfer due to roughness is far greater for Ucon than it is for water. These findings are supported by boundary-layer theory but not by most empirical equations proposed to date. In 1952, Sams (*5*) developed an equation for heat transfer to low Prandtl number fluids in rough pipes.

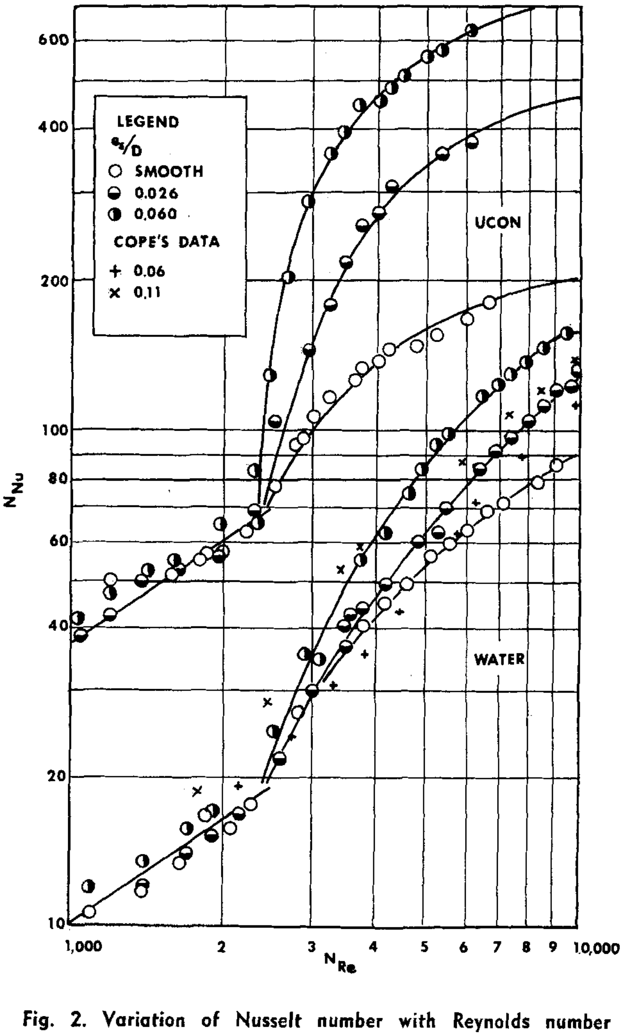

Fig. 2. Variation of Nusselt number with Reynolds number for water and Ucon in smooth and rough pipes.

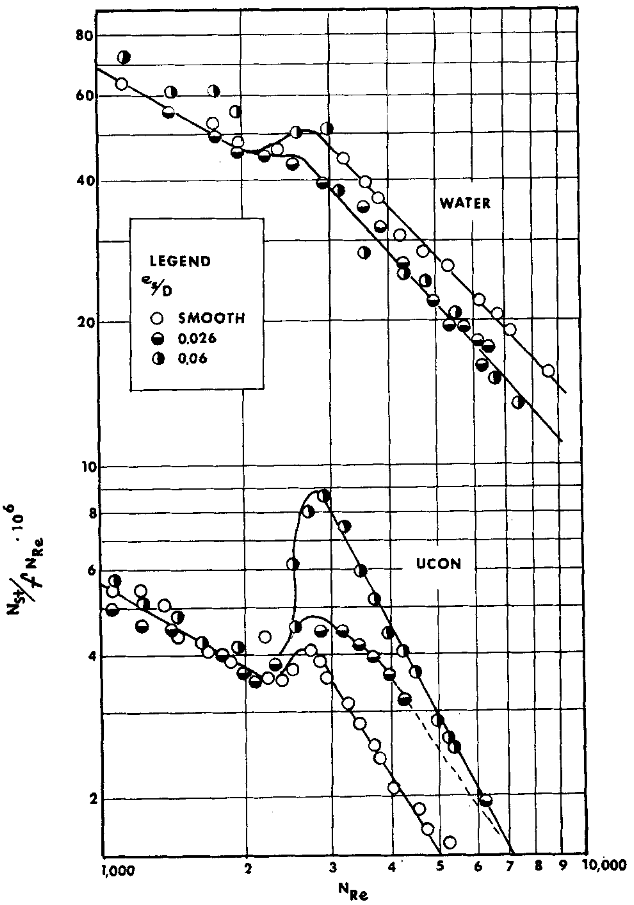

Fig. 3. Heat transfer efficiency for smooth and rough pipes.

His equation does not provide the Stanton number with a Prandtl number dependence. Consequently, the present experimental results for Ucon are an order of magnitude smaller than the predicted results of Sams' equation.

Heat transfer efficiency can be represented by the dimensionless group $N_{St}/f\,N_{Re}$, which is equivalent to the heat transmission per unit pressure drop. Figure 3 shows the variation of this group with the Reynolds number for all the tubes and fluids used in the investigation. As reported previously (*1* to *3*), the smooth tube was more efficient than either of the rough tubes when water was used as the test fluid. However, in the case of Ucon, the rough tubes are significantly more efficient in the transition-turbulent region. This difference in behavior holds whether the true surface area or the effective smooth surface is used to calculate the heat transfer coefficient. The increase in surface area for the roughest tube does not exceed 28%, whereas the increase in efficiency over that in the smooth tube is never less than 200% in the region studied.

It can be shown from boundary-layer theory that a smooth tube will eventually become more efficient than a rough one as the Reynolds number is increased, regardless of the Prandtl number of the fluid. However, the larger the Prandtl number the higher the Reynolds number must be in order for this to happen. Therefore in practice this fact is not important because very high Reynolds numbers are exceedingly difficult to obtain with viscous and consequently high-Prandtl-number fluids.

The work is now being extended to cover a wider range of Reynolds and Prandtl numbers in tubes with different roughness patterns.

NOTATION

f = friction factor
e_s/D = equivalent sand roughness
e/D = measured roughness
N_{Nu} = Nusselt number
N_{Pr} = Prandtl number
N_{Re} = Reynolds number
N_{St} = $h/\rho u c_p$ = Stanton number

LITERATURE CITED

1. Dipprey, D. F., and R. H. Sabersky, *Intern. J. Heat Mass Transfer,* **6,** 329-353 (1963).
2. Cope, W. F., *Proc. Inst. Mech. Engrs.,* **145,** 99 (1941).
3. Hastrup, R. C., *Jet Propulsion,* 259 (April, 1958).
4. McAdams, W. H., "Heat Transmission," p. 241, McGraw-Hill, New York (1954).
5. Sams, E. W., *Natl. Advisory Committee Aeron. RM E52 D17* (1952).
6. Perry, R. H., "Chemical Engineers' Handbook," 4 ed., pp. 5-20, McGraw-Hill, New York (1963).

Diffusion with Consecutive Heterogeneous Reactions

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Irreversible first-order heterogeneous chemical reaction on the wall of a tube in which a fluid is flowing in fully developed laminar flow has been studied by several investigators (*1, 2*). In this note the problem will be extended to the case of consecutive reactions. The analysis parallels that of Wissler and Schechter (*3*), who treated consecutive irreversible first-order homogeneous reactions.

Consider the heterogeneous reactions

$$A_1 \rightarrow \beta A_2 \rightarrow \text{products} \qquad (1)$$

The concentrations are taken to be dilute and fluid properties are assumed constant. A plug flow velocity profile will be used, although a similar treatment could be used for the Poiseuille distribution. Under these restrictions the dimensionless material balance on component 1 is

$$\frac{1}{y}\frac{\partial}{\partial y}y\frac{\partial C_1}{\partial y} = \frac{\partial C_1}{\partial x} \qquad (2)$$

with boundary conditions

$x = 0$ $\qquad C_1 = 1$
$y = 0$ $\qquad C_1 =$ finite
$y = 1$ $\qquad \dfrac{\partial C_1}{\partial y} = -K_1C_1$

The solution to this system is

$$C_1 = \sum_i a_i e^{-\lambda_i^2 x} J_0(\lambda_i y) \qquad (3)$$

where